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Complementarity in multiple beam interference

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Abstract
We show that for multipath quantum interferometers the visibility of the interference and 'which-path' information are not always complementary observables. This implies that there are path measurements that do not destroy the interference.

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Complementarity is at the conceptual heart of quantum theory. This concept has no classical analogue and implies that quantum systems possess properties that are mutually exclusive: the observation of one of them precludes the observation of the other. Perhaps the best illustration of this idea is the interferometric wave–particle duality. If a which-path detector is arranged in order to determine the path taken by the particle within the interferometer, the interference is necessarily destroyed. The origin and implications of complementarity have been extensively discussed since the early stages of quantum theory until the present day. Nevertheless, subtle tests of complementarity recently introduced and the vivid debate that followed them reveal that this issue is not fully understood yet [1–3].

In this sense, an intriguing experiment has been put forward very recently that seemingly defies basic quantum intuition based on the idea of complementarity. It has been demonstrated that the detection of the path followed by an interfering particle in a multipath interferometer can increase the fringe contrast [4].

In this work we solve this paradox by showing that path and visibility in multiple beam interference are not necessarily complementary observables. In such a case the standard quantum arguments based on complementarity do not apply.

In order to be specific we consider the same multipath arrangement carried out experimentally in [4]. The Hilbert space for the system is spanned by N orthogonal vectors $|n\rangle$, $n = 1, \ldots, N$, so that a general pure state $|\psi\rangle$ can be expressed as

$$|\psi\rangle = \sum_{n=1}^{N} c_n |n\rangle. \tag{1}$$

In the experimental arrangement of [4] the vectors $|n\rangle$ are the magnetic sublevels of the $F = 3$ hyperfine component of the caesium electronic ground state.

Within this finite-dimensional system, an interferometer is constructed as follows. The atom is initially prepared in a nonabsorbing dark state $|\psi\rangle$ of the form (1). Then the system experiences a phase shift of the form

$$|\psi_{\rm out}\rangle = {\rm e}^{{\rm i}\varphi\hat{n}}|\psi\rangle \tag{2}$$

where

$$\hat{n} = \sum_{n=1}^{N} n|n\rangle\langle n|. \tag{3}$$

Finally, it is detected whether the state $|\psi_{\rm out}\rangle$ is in the dark state $|\psi\rangle$ or not. This is achieved by suitably illuminating the atom and detecting the fluorescence emitted. The probability that no fluorescence photons are emitted is, as a function of φ,

$$P(\varphi) = |\langle\psi|\psi_{\rm out}\rangle|^2 = |\langle\psi|{\rm e}^{{\rm i}\varphi\hat{n}}|\psi\rangle|^2. \tag{4}$$

This scheme can actually be regarded as a multiple beam interferometer where φ is the phase difference between adjacent paths and $P(\varphi)$ represents the interference pattern. As a quantitative measure of the degree of interference we may use the usual fringe contrast or visibility defined as

$$\mathcal{V} = \frac{I_{\rm max} - I_{\rm min}}{I_{\rm max} + I_{\rm min}} \tag{5}$$

where $I_{\rm max}$ and $I_{\rm min}$ denote the maxima and minima values of $P(\varphi)$. In what follows we denote by φ the wave-like variable measured in this arrangement.

On the other hand, the experimental path detection carried out in [4] detects whether the particle follows one of the possible paths, say $|N\rangle$, or not. The measurement has only two outcomes (yes or no) and is fully described by the two orthogonal projectors

$$\Delta(N) = |N\rangle\langle N| \qquad \Delta(\neg N) = \sum_{n\neq N} |n\rangle\langle n| \tag{6}$$

so that the probability that the particle follows the path $|N\rangle$ is $P(N) = \langle\psi|\Delta(N)|\psi\rangle = 1 - P(\neg N)$. We denote by N the particle-like observable defined by this measuring strategy.

Before examining whether φ and N are complementary, let us point out that $P(\varphi)$ is not a standard quantum probability distribution for the φ variable. As a matter of fact, $\langle\psi|{\rm e}^{{\rm i}\varphi\hat{n}}|\psi\rangle$ is the characteristic function for the operator $\hat{n}$ [5]. In the first place we have that $P(\varphi)$ is not normalized $\int_{2\pi} {\rm d}\varphi\, P(\varphi) \neq 1$. Moreover, $P(\varphi)$ is not linear on the density matrix $|\psi\rangle\langle\psi|$. This means that there is no positive operator measure $\Delta(\varphi)$ such that

$$P(\varphi) = \langle\psi|\Delta(\varphi)|\psi\rangle \tag{7}$$

for every system state $|\psi\rangle$. Nevertheless, $P(\varphi)$ can be recast into standard quantum mechanics after a suitable normalization:

$$\tilde{P}(\varphi) = \frac{1}{\int_{2\pi} {\rm d}\varphi'\, P(\varphi')} P(\varphi) = |\langle\varphi|\tilde{\psi}\rangle|^2 \tag{8}$$

where the effective system state $|\tilde{\psi}\rangle$ is defined as

$$|\tilde{\psi}\rangle = \frac{1}{\sqrt{\sum_{n'=1}^{N} |c_{n'}|^4}} \sum_{n=1}^{N} |c_n|^2\, |n\rangle \tag{9}$$

and the unnormalized and nonorthogonal states

$$|\varphi\rangle = \frac{1}{\sqrt{2\pi}} \sum_{n=1}^{N} e^{-in\varphi} |n\rangle \tag{10}$$

are the phase states for a finite-dimensional system [6].

At this stage it is worth pointing out that equations (8) and (10) reveal that the arrangement we are considering here is a direct measurement of the phase for the effective state $|\tilde{\psi}\rangle$ [6]. This is a very remarkable result since feasible measurements of phase observables are very rare.

After these definitions we can show that φ and N are not complementary observables for $N > 2$. Two observables are complementary if precise knowledge of one of them implies that all possible outcomes of measuring the other one are equally probable [1]. This objective can be accomplished by proposing a pair of counter-examples.

As the first counter-example let us consider the case $N > 2$ and the state

$$|\psi\rangle = c_1|1\rangle + c_2|2\rangle \tag{11}$$

which has a well defined value of the observable N for every c_1, c_2 since $P(N) = 0$ and $P(\neg N) = 1$. If N and φ were complementary $P(\varphi)$ should be constant for the state (11), i.e. $\tilde{P}(\varphi) = 1/(2\pi)$, and, in particular $\mathcal{V} = 0$. However, from equation (4) we have

$$P(\varphi) = |c_1|^4 + |c_2|^4 + 2|c_1c_2|^2 \cos\varphi \tag{12}$$

and when $c_1c_2 \neq 0$ the outcomes of the measurement of φ are not equally probable. Moreover, if $|c_1| = |c_2|$ the visibility is maximum $\mathcal{V} = 1$.

As a further counter-example we can consider that the system is in a (suitably normalized) phase state (10), $|\psi\rangle \propto |\varphi_0\rangle$, for some constant phase value φ_0. From equation (4) the distribution $P(\varphi)$ for this state is

$$P(\varphi) = \frac{1}{N^2}\left[\frac{\sin(N\varphi/2)}{\sin(\varphi/2)}\right]^2 \tag{13}$$

which has maximum visibility $\mathcal{V} = 1$. This is formally equivalent to the interference pattern produced by a diffraction grating with N slits. On the other hand, for this state the path probability defined by equation (6) is

$$P(\neg N) = 1 - \frac{1}{N} \qquad P(N) = \frac{1}{N}. \tag{14}$$

Therefore, for $N > 2$ we have that $P(\neg N) \neq P(N)$ and the two possible outcomes of the measurement of N have different probabilities.

Since φ and N are not complementary the path measurement represented by the observable N does not necessarily imply the destruction of the interference, as it has been experimentally demonstrated in [4]. We stress that in order to obtain this result it is crucial that the dimension of the system space is larger than two. Otherwise, for a two-dimensional system, φ and N are complementary and path detection unavoidably destroys interference via phase randomization [3].

This result does not mean that there are no phase observables complementary to N. For example we can define the positive operator measure

$$\Delta(\phi) = \frac{1}{2\pi}\left(1 + e^{i\phi}|N\rangle\langle N_\perp| + e^{-i\phi}|N_\perp\rangle\langle N|\right) \tag{15}$$

where $|N_\perp\rangle$ is any vector orthogonal to $|N\rangle$. It can be easily seen that for any state with well defined N (i.e. $|\langle N|\psi\rangle| = 1$ or 0) the probability distribution for ϕ is uniform

$P(\phi) = \langle\psi|\Delta(\phi)|\psi\rangle = 1/(2\pi)$. This observable is closely related to the vector phase states studied in [7]. We can also show that φ has complementary observables. This is the case of the observable n defined by the projectors $|n\rangle\langle n|$ since for the phase states (10) we have $|\langle n|\varphi\rangle| = \text{constant}$.

Summarizing, we have shown that path and visibility are not always complementary variables in multipath quantum interferometers. Then, nothing prevents path measurements increasing the visibility of the interference.

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